

# Tuning Four Pitches

Just intonation occurs when all the pitches of a chord are tuned to be harmonics of a common fundamental frequency. Tuning in this way makes a chord “ring” without any beats. The chart below reveals how to tune many common four pitch chords without any pitch drift. It achieves this by preserving the intonation of the reinforced fundamental of the chord. It lists the **closest pitch set**, **tuned pitches**, **pitch adjustments**, and **tonalness values** for the most tunable four-pitch harmonic states.

A **harmonic state** is a set of partial numbers above a common fundamental frequency. For example, the harmonic state {4,5,6,7} represents the fourth, fifth, sixth, and seventh harmonics of a given fundamental. In other words, the harmonic state {4,5,6,7} is a justly tuned dominant seventh chord.

The **closest pitch set** is a set of semitone values that best approximates the harmonic state with pitches from the equal tempered chromatic scale. For example, the pitch set {0,4,7,10} represents three pitches which are four, seven, and ten semitones above a fourth lowest pitch. In other words, the pitch set {0,4,7,10} is an equal tempered dominant seventh chord.

The **tuned pitches** are a set of semitone values which represent how to properly tune the chord and preserve its reinforced fundamental.

The **pitch adjustments** are simply the difference between the tuned pitches and the closest pitch set. They reveal how to tune each of the pitches of the chord relative to equal temperament.

The **tonalness values** reveal the strength of the reinforced fundamental frequency. For this chart, the tonalness calculation assumes that each component pitch of the chord has an amplitude equal to 1. A higher tonalness value means that the chord will “ring” more if it is properly tuned.

Harmonic State	Closest Pitch Set	Tuned First Pitch	Tuned Second Pitch	Tuned Third Pitch	Tuned Fourth Pitch	First Pitch Adjustment	Second Pitch Adjustment	Third Pitch Adjustment	Fourth Pitch Adjustment	Tonalness
{4, 5, 6, 7}	{0, 4, 7, 10}	0.0904	3.9535	7.1099	9.7786	0.0904	-0.0465	0.1099	-0.2214	0.7595
{5, 6, 7, 8}	{0, 3, 6, 8}	-0.0287	3.1277	5.7964	8.1082	-0.0287	0.1277	-0.2036	0.1082	0.6345
{5, 6, 7, 9}	{0, 3, 6, 10}	-0.0333	3.1232	5.7919	10.1427	-0.0333	0.1232	-0.2081	0.1427	0.6206
{5, 6, 8, 9}	{0, 3, 8, 10}	-0.1041	3.0523	8.0328	10.0719	-0.1041	0.0523	0.0328	0.0719	0.6028
{5, 7, 8, 9}	{0, 6, 8, 10}	-0.0202	5.8050	8.1167	10.1558	-0.0202	-0.1950	0.1167	0.1558	0.5790
{6, 7, 8, 9}	{0, 3, 5, 7}	0.0872	2.7559	5.0677	7.1068	0.0872	-0.2441	0.0677	0.1068	0.5456
{6, 7, 8, 10}	{0, 3, 5, 9}	0.1224	2.7911	5.1028	8.9660	0.1224	-0.2089	0.1028	-0.0340	0.5345
{6, 7, 8, 11}	{0, 3, 5, 11}	0.0093	2.6780	4.9898	10.5029	0.0093	-0.3220	-0.0102	-0.4971	0.5254
{6, 7, 9, 10}	{0, 3, 7, 9}	0.1168	2.7855	7.1363	8.9604	0.1168	-0.2145	0.1363	-0.0396	0.5206
{6, 7, 9, 11}	{0, 3, 7, 10}	0.0005	2.6693	7.0201	10.4942	0.0005	-0.3307	0.0201	0.4942	0.5115
{6, 8, 9, 10}	{0, 5, 7, 9}	0.0316	5.0121	7.0512	8.8752	0.0316	0.0121	0.0512	-0.1248	0.5028
{6, 7, 10, 11}	{0, 3, 9, 11}	0.0362	2.7049	8.8797	10.5298	0.0362	-0.2951	-0.1203	-0.4702	0.5004
{6, 8, 9, 11}	{0, 5, 7, 10}	-0.0903	4.8901	6.9292	10.4033	-0.0903	-0.1099	-0.0708	0.4033	0.4937
{6, 8, 10, 11}	{0, 5, 9, 10}	-0.0555	4.9249	8.7881	10.4381	-0.0555	-0.0751	-0.2119	0.4381	0.4826
{7, 8, 9, 10}	{0, 2, 4, 6}	-0.1993	2.1125	4.1516	5.9756	-0.1993	0.1125	0.1516	-0.0244	0.4790
{7, 8, 9, 11}	{0, 2, 4, 8}	-0.1320	2.1797	4.2188	7.6929	-0.1320	0.1797	0.2188	-0.3071	0.4699
{6, 9, 10, 11}	{0, 7, 9, 10}	-0.0670	6.9525	8.7766	10.4266	-0.0670	-0.0475	-0.2234	0.4266	0.4687
{7, 8, 9, 12}	{0, 2, 4, 9}	-0.2283	2.0834	4.1225	9.1030	-0.2283	0.0834	0.1225	0.1030	0.4623
{7, 8, 10, 11}	{0, 2, 6, 8}	-0.0884	2.2234	6.0865	7.7366	-0.0884	0.2234	0.0865	-0.2634	0.4588
{7, 8, 9, 13}	{0, 2, 4, 11}	-0.1232	2.1885	4.2276	10.5938	-0.1232	0.1885	0.2276	-0.4062	0.4559
{7, 8, 10, 12}	{0, 2, 6, 9}	-0.1863	2.1254	5.9886	9.1450	-0.1863	0.1254	-0.0114	0.1450	0.4512
{7, 9, 10, 11}	{0, 4, 6, 8}	-0.0912	4.2597	6.0837	7.7338	-0.0912	0.2597	0.0837	-0.2662	0.4449
{7, 8, 10, 13}	{0, 2, 6, 11}	-0.0780	2.2338	6.0969	10.6390	-0.0780	0.2338	0.0969	-0.3610	0.4448
{7, 8, 11, 12}	{0, 2, 8, 9}	-0.1146	2.1972	7.7103	9.2167	-0.1146	0.1972	-0.2897	0.2167	0.4421
{7, 9, 10, 12}	{0, 4, 6, 9}	-0.1923	4.1586	5.9826	9.1390	-0.1923	0.1586	-0.0174	0.1390	0.4373

Harmonic State	Closest Pitch Set	Tuned First Pitch	Tuned Second Pitch	Tuned Third Pitch	Tuned Fourth Pitch	First Pitch Adjustment	Second Pitch Adjustment	Third Pitch Adjustment	Fourth Pitch Adjustment	Tonalness
{7, 8, 11, 13}	{0, 2, 8, 11}	-0.0029	2.3088	7.8220	10.7141	-0.0029	0.3088	-0.1780	-0.2859	0.4357
{7, 9, 10, 13}	{0, 4, 6, 11}	-0.0805	4.2703	6.0943	10.6365	-0.0805	0.2703	0.0943	-0.3635	0.4309
{7, 9, 11, 12}	{0, 4, 8, 9}	-0.1183	4.2325	7.7066	9.2130	-0.1183	0.2325	-0.2934	0.2130	0.4282
{7, 8, 12, 13}	{0, 2, 9, 11}	-0.1047	2.2071	9.2266	10.6124	-0.1047	0.2071	0.2266	-0.3876	0.4281
{8, 9, 10, 11}	{0, 2, 4, 6}	0.1255	2.1646	3.9887	5.6387	0.1255	0.1646	-0.0113	-0.3613	0.4270
{7, 9, 11, 13}	{0, 4, 8, 11}	-0.0031	4.3478	7.8218	10.7139	-0.0031	0.3478	-0.1782	-0.2861	0.4218
{8, 9, 10, 12}	{0, 2, 4, 7}	0.0184	2.0575	3.8815	7.0379	0.0184	0.0575	-0.1185	0.0379	0.4194
{7, 10, 11, 12}	{0, 6, 8, 9}	-0.0700	6.1049	7.7550	9.2613	-0.0700	0.1049	-0.2450	0.2613	0.4171
{7, 9, 12, 13}	{0, 4, 9, 11}	-0.1082	4.2426	9.2231	10.6088	-0.1082	0.2426	0.2231	-0.3912	0.4142
{8, 9, 10, 13}	{0, 2, 4, 8}	-0.0529	1.9862	3.8103	8.3524	-0.0529	-0.0138	-0.1897	0.3524	0.4130
{7, 10, 11, 13}	{0, 6, 8, 11}	0.0492	6.2241	7.8741	10.7662	0.0492	0.2241	-0.1259	-0.2338	0.4107
{8, 9, 11, 12}	{0, 2, 6, 7}	0.0933	2.1324	5.6065	7.1128	0.0933	0.1324	-0.3935	0.1128	0.4104
{8, 9, 10, 14}	{0, 2, 4, 10}	0.0776	2.1167	3.9407	9.7658	0.0776	0.1167	-0.0593	-0.2342	0.4075
{8, 9, 11, 13}	{0, 2, 6, 8}	0.0216	2.0607	5.5348	8.4269	0.0216	0.0607	-0.4652	0.4269	0.4039
{7, 10, 12, 13}	{0, 6, 9, 11}	-0.0579	6.1170	9.2734	10.6591	-0.0579	0.1170	0.2734	-0.3409	0.4031
{8, 9, 10, 15}	{0, 2, 4, 11}	0.0426	2.0817	3.9057	10.9253	0.0426	0.0817	-0.0943	-0.0747	0.4028
{8, 10, 11, 12}	{0, 4, 6, 7}	0.1411	4.0042	5.6542	7.1606	0.1411	0.0042	-0.3458	0.1606	0.3992
{8, 9, 11, 14}	{0, 2, 6, 10}	0.1561	2.1952	5.6692	9.8443	0.1561	0.1952	-0.3308	-0.1557	0.3984
{8, 9, 12, 13}	{0, 2, 7, 8}	-0.0937	1.9454	6.9258	8.3116	-0.0937	-0.0546	-0.0742	0.3116	0.3964
{7, 11, 12, 13}	{0, 8, 9, 11}	0.0256	7.8505	9.3569	10.7426	0.0256	-0.1495	0.3569	-0.2574	0.3940
{8, 9, 11, 15}	{0, 2, 6, 11}	0.1212	2.1603	5.6344	11.0039	0.1212	0.1603	-0.3656	0.0039	0.3937
{8, 10, 11, 13}	{0, 4, 6, 8}	0.0681	3.9313	5.5813	8.4734	0.0681	-0.0687	-0.4187	0.4734	0.3928
{8, 9, 12, 14}	{0, 2, 7, 10}	0.0417	2.0808	7.0612	9.7299	0.0417	0.0808	0.0612	-0.2701	0.3909
{8, 10, 11, 14}	{0, 4, 6, 10}	0.2071	4.0702	5.7203	9.8953	0.2071	0.0702	-0.2797	-0.1047	0.3873
{8, 9, 12, 15}	{0, 2, 7, 11}	0.0048	2.0439	7.0243	10.8875	0.0048	0.0439	0.0243	-0.1125	0.3861
{9, 10, 11, 12}	{0, 2, 3, 5}	-0.0620	1.7621	3.4121	4.9185	-0.0620	-0.2379	0.4121	-0.0815	0.3854
{8, 10, 12, 13}	{0, 4, 7, 8}	-0.0496	3.8135	6.9699	8.3557	-0.0496	-0.1865	-0.0301	0.3557	0.3853
{8, 9, 13, 14}	{0, 2, 8, 10}	-0.0345	2.0046	8.3708	9.6538	-0.0345	0.0046	0.3708	-0.3462	0.3845
{8, 10, 11, 15}	{0, 4, 6, 11}	0.1719	4.0350	5.6851	11.0546	0.1719	0.0350	-0.3149	0.0546	0.3826
{8, 9, 13, 15}	{0, 2, 8, 11}	-0.0729	1.9662	8.3323	10.8097	-0.0729	-0.0338	0.3323	-0.1903	0.3797
{9, 10, 11, 13}	{0, 2, 3, 6}	-0.1416	1.6824	3.3325	6.2245	-0.1416	-0.3176	0.3325	0.2245	0.3789
{8, 11, 12, 13}	{0, 6, 7, 8}	0.0304	5.5436	7.0500	8.4357	0.0304	-0.4564	0.0500	0.4357	0.3762
{8, 10, 12, 15}	{0, 4, 7, 11}	0.0530	3.9161	7.0726	10.9357	0.0530	-0.0839	0.0726	-0.0643	0.3750
{8, 9, 14, 15}	{0, 2, 10, 11}	0.0688	2.1079	9.7571	10.9515	0.0688	0.1079	-0.2429	-0.0485	0.3742
{9, 10, 11, 14}	{0, 2, 3, 8}	-0.0012	1.8229	3.4729	7.6480	-0.0012	-0.1771	0.4729	-0.3520	0.3734
{8, 10, 13, 14}	{0, 4, 8, 10}	0.0128	3.8759	8.4181	9.7011	0.0128	-0.1241	0.4181	-0.2989	0.3734
{9, 10, 12, 13}	{0, 2, 5, 6}	-0.0241	1.8000	4.9564	6.3421	-0.0241	-0.2000	-0.0436	0.3421	0.3714
{8, 11, 12, 14}	{0, 6, 7, 10}	0.1751	5.6883	7.1946	9.8633	0.1751	-0.3117	0.1946	-0.1367	0.3707
{9, 10, 11, 15}	{0, 2, 3, 9}	-0.0409	1.7832	3.4332	8.8027	-0.0409	-0.2168	0.4332	-0.1973	0.3687
{8, 10, 13, 15}	{0, 4, 8, 11}	-0.0262	3.8369	8.3790	10.8565	-0.0262	-0.1631	0.3790	-0.1435	0.3686
{8, 11, 12, 15}	{0, 6, 7, 11}	0.1379	5.6511	7.1574	11.0206	0.1379	-0.3489	0.1574	0.0206	0.3659
{9, 10, 12, 14}	{0, 2, 5, 8}	0.1210	1.9451	5.1015	7.7702	0.1210	-0.0549	0.1015	-0.2298	0.3659
{9, 10, 11, 16}	{0, 2, 3, 10}	-0.0633	1.7608	3.4108	9.8976	-0.0633	-0.2392	0.4108	-0.1024	0.3645
{8, 11, 13, 14}	{0, 6, 9, 10}	0.0970	5.6102	8.5023	9.7853	0.0970	-0.3898	-0.4977	-0.2147	0.3643
{8, 10, 14, 15}	{0, 4, 10, 11}	0.1206	3.9837	9.8088	11.0032	0.1206	-0.0163	-0.1912	0.0032	0.3631
{9, 11, 12, 13}	{0, 3, 5, 6}	-0.1922	3.2819	4.7882	6.1740	-0.1922	0.2819	-0.2118	0.1740	0.3623

Harmonic State	Closest Pitch Set	Tuned First Pitch	Tuned Second Pitch	Tuned Third Pitch	Tuned Fourth Pitch	First Pitch Adjustment	Second Pitch Adjustment	Third Pitch Adjustment	Fourth Pitch Adjustment	Tonalness
{9, 10, 12, 15}	{0, 2, 5, 9}	0.0821	1.9062	5.0626	8.9257	0.0821	-0.0938	0.0626	-0.0743	0.3611
{9, 10, 11, 17}	{0, 2, 3, 11}	-0.0724	1.7517	3.4017	10.9381	-0.0724	-0.2483	0.4017	-0.0619	0.3608
{8, 11, 13, 15}	{0, 6, 8, 11}	0.0581	5.5713	8.4634	10.9408	0.0581	-0.4287	0.4634	-0.0592	0.3595
{9, 10, 13, 14}	{0, 2, 6, 8}	0.0403	1.8643	6.4065	7.6895	0.0403	-0.1357	0.4065	-0.3105	0.3595
{9, 10, 12, 16}	{0, 2, 5, 10}	0.0607	1.8847	5.0412	10.0216	0.0607	-0.1153	0.0412	0.0216	0.3569
{9, 11, 12, 14}	{0, 3, 5, 8}	-0.0460	3.4281	4.9345	7.6032	-0.0460	0.4281	-0.0655	-0.3968	0.3568
{8, 12, 13, 14}	{0, 7, 8, 10}	-0.0295	6.9900	8.3757	9.6587	-0.0295	-0.0100	0.3757	-0.3413	0.3567
{9, 10, 13, 15}	{0, 2, 6, 9}	-0.0004	1.8236	6.3658	8.8432	-0.0004	-0.1764	0.3658	-0.1568	0.3547
{8, 11, 14, 15}	{0, 6, 10, 11}	0.2100	5.7232	9.8983	11.0927	0.2100	-0.2768	-0.1017	0.0927	0.3540
{9, 10, 12, 17}	{0, 2, 5, 11}	0.0527	1.8767	5.0331	11.0631	0.0527	-0.1233	0.0331	0.0631	0.3533
{9, 11, 12, 15}	{0, 3, 5, 9}	-0.0882	3.3859	4.8923	8.7554	-0.0882	0.3859	-0.1077	-0.2446	0.3520
{8, 12, 13, 15}	{0, 7, 8, 11}	-0.0710	6.9486	8.3343	10.8117	-0.0710	-0.0514	0.3343	-0.1883	0.3519
{10, 11, 12, 13}	{0, 2, 3, 5}	0.1538	1.8038	3.3102	4.6959	0.1538	-0.1962	0.3102	-0.3041	0.3512
{9, 10, 13, 16}	{0, 2, 6, 10}	-0.0232	1.8009	6.3430	9.9377	-0.0232	-0.1991	0.3430	-0.0623	0.3505
{9, 11, 13, 14}	{0, 3, 6, 8}	-0.1319	3.3422	6.2343	7.5173	-0.1319	0.3422	0.2343	-0.4827	0.3504
{9, 10, 14, 15}	{0, 2, 8, 9}	0.1520	1.9761	7.8012	8.9956	0.1520	-0.0239	-0.1988	-0.0044	0.3492
{9, 11, 12, 16}	{0, 3, 5, 10}	-0.1122	3.3619	4.8683	9.8487	-0.1122	0.3619	-0.1317	-0.1513	0.3479
{9, 10, 13, 17}	{0, 2, 6, 11}	-0.0322	1.7918	6.3339	10.9782	-0.0322	-0.2082	0.3339	-0.0218	0.3469
{8, 12, 14, 15}	{0, 7, 10, 11}	0.0821	7.1017	9.7704	10.9648	0.0821	0.1017	-0.2296	-0.0352	0.3464
{10, 11, 12, 14}	{0, 2, 3, 6}	0.0905	1.7405	3.2469	5.9156	0.0905	-0.2595	0.2469	-0.0844	0.3457
{9, 11, 13, 15}	{0, 3, 6, 9}	-0.1760	3.2980	6.1901	8.6676	-0.1760	0.2980	0.1901	-0.3324	0.3456
{9, 10, 14, 16}	{0, 2, 8, 10}	0.1307	1.9547	7.7799	10.0916	0.1307	-0.0453	-0.2201	0.0916	0.3450
{9, 11, 12, 17}	{0, 3, 5, 11}	-0.1223	3.3518	4.8582	10.8882	-0.1223	0.3518	-0.1418	-0.1118	0.3442
{9, 12, 13, 14}	{0, 5, 6, 8}	-0.0043	4.9761	6.3619	7.6448	-0.0043	-0.0239	0.3619	-0.3552	0.3428
{9, 11, 13, 16}	{0, 3, 6, 10}	-0.2016	3.2725	6.1646	9.7593	-0.2016	0.2725	0.1646	-0.2407	0.3414
{9, 10, 14, 17}	{0, 2, 8, 11}	0.1232	1.9472	7.7723	11.1336	0.1232	-0.0528	-0.2277	0.1336	0.3414
{10, 11, 12, 15}	{0, 2, 3, 7}	0.0513	1.7013	3.2077	7.0708	0.0513	-0.2987	0.2077	0.0708	0.3409
{9, 10, 15, 16}	{0, 2, 9, 10}	0.0895	1.9136	8.9331	10.0504	0.0895	-0.0864	-0.0669	0.0504	0.3403
{9, 11, 14, 15}	{0, 3, 8, 9}	-0.0224	3.4517	7.6268	8.8212	-0.0224	0.4517	-0.3732	-0.1788	0.3401
{8, 13, 14, 15}	{0, 8, 10, 11}	-0.0032	8.4021	9.6851	10.8795	-0.0032	0.4021	-0.3149	-0.1205	0.3400
{10, 11, 13, 14}	{0, 2, 5, 6}	0.2344	1.8845	4.7765	6.0595	0.2344	-0.1155	-0.2235	0.0595	0.3393
{9, 12, 13, 15}	{0, 5, 6, 9}	-0.0477	4.9328	6.3185	8.7959	-0.0477	-0.0672	0.3185	-0.2041	0.3380
{9, 11, 13, 17}	{0, 3, 6, 11}	-0.2128	3.2613	6.1534	10.7976	-0.2128	0.2613	0.1534	-0.2024	0.3378
{10, 11, 12, 16}	{0, 2, 3, 8}	0.0304	1.6804	3.1868	8.1672	0.0304	-0.3196	0.1868	0.1672	0.3367
{9, 10, 15, 17}	{0, 2, 9, 11}	0.0814	1.9055	8.9250	11.0919	0.0814	-0.0945	-0.0750	0.0919	0.3366
{9, 11, 14, 16}	{0, 3, 8, 10}	-0.0464	3.4277	7.6027	9.9145	-0.0464	0.4277	-0.3973	-0.0855	0.3359
{10, 11, 13, 15}	{0, 2, 5, 7}	0.1965	1.8465	4.7386	7.2161	0.1965	-0.1535	-0.2614	0.2161	0.3345
{9, 12, 13, 16}	{0, 5, 6, 10}	-0.0722	4.9083	6.2940	9.8887	-0.0722	-0.0917	0.2940	-0.1113	0.3339
{10, 11, 12, 17}	{0, 2, 3, 9}	0.0235	1.6735	3.1799	9.2099	0.0235	-0.3265	0.1799	0.2099	0.3331
{9, 12, 14, 15}	{0, 5, 8, 9}	0.1116	5.0921	7.7608	8.9552	0.1116	0.0921	-0.2392	-0.0448	0.3325
{9, 10, 16, 17}	{0, 2, 10, 11}	0.0584	1.8825	10.0193	11.0689	0.0584	-0.1175	0.0193	0.0689	0.3324
{9, 11, 14, 17}	{0, 3, 8, 11}	-0.0561	3.4179	7.5930	10.9543	-0.0561	0.4179	-0.4070	-0.0457	0.3323
{10, 12, 13, 14}	{0, 3, 5, 6}	0.1045	3.2610	4.6467	5.9297	0.1045	0.2610	-0.3533	-0.0703	0.3317
{9, 11, 15, 16}	{0, 3, 9, 10}	-0.0913	3.3828	8.7523	9.8696	-0.0913	0.3828	-0.2477	-0.1304	0.3312
{10, 11, 13, 16}	{0, 2, 5, 8}	0.1770	1.8271	4.7192	8.3139	0.1770	-0.1729	-0.2808	0.3139	0.3303
{9, 12, 13, 17}	{0, 5, 6, 11}	-0.0822	4.8982	6.2839	10.9282	-0.0822	-0.1018	0.2839	-0.0718	0.3302

Harmonic State	Closest Pitch Set	Tuned First Pitch	Tuned Second Pitch	Tuned Third Pitch	Tuned Fourth Pitch	First Pitch Adjustment	Second Pitch Adjustment	Third Pitch Adjustment	Fourth Pitch Adjustment	Tonalness
{10, 11, 12, 18}	{0, 2, 3, 10}	0.0273	1.6773	3.1837	10.2033	0.0273	-0.3227	0.1837	0.2033	0.3298
{10, 11, 14, 15}	{0, 2, 6, 7}	0.1307	1.7807	5.9558	7.1503	0.1307	-0.2193	-0.0442	0.1503	0.3290
{9, 12, 14, 16}	{0, 5, 8, 10}	0.0887	5.0692	7.7379	10.0496	0.0887	0.0692	-0.2621	0.0496	0.3284
{9, 11, 15, 17}	{0, 3, 9, 11}	-0.1016	3.3724	8.7420	10.9088	-0.1016	0.3724	-0.2580	-0.0912	0.3275
{10, 12, 13, 15}	{0, 3, 5, 7}	0.0639	3.2203	4.6060	7.0834	0.0639	0.2203	-0.3940	0.0834	0.3269
{10, 11, 12, 19}	{0, 2, 3, 11}	0.0394	1.6895	3.1958	11.1514	0.0394	-0.3105	0.1958	0.1514	0.3269
{10, 11, 13, 17}	{0, 2, 5, 9}	0.1716	1.8217	4.7138	9.3581	0.1716	-0.1783	-0.2862	0.3581	0.3267
{9, 13, 14, 15}	{0, 6, 8, 9}	0.0224	6.3886	7.6716	8.8660	0.0224	0.3886	-0.3284	-0.1340	0.3261
{10, 11, 14, 16}	{0, 2, 6, 8}	0.1101	1.7601	5.9352	8.2469	0.1101	-0.2399	-0.0648	0.2469	0.3248
{9, 12, 14, 17}	{0, 5, 8, 11}	0.0803	5.0608	7.7295	11.0908	0.0803	0.0608	-0.2705	0.0908	0.3247
{9, 12, 15, 16}	{0, 5, 9, 10}	0.0448	5.0253	8.8884	10.0057	0.0448	0.0253	-0.1116	0.0057	0.3236
{10, 11, 13, 18}	{0, 2, 5, 10}	0.1771	1.8271	4.7192	10.3530	0.1771	-0.1729	-0.2808	0.3530	0.3234
{9, 11, 16, 17}	{0, 3, 10, 11}	-0.1276	3.3464	9.8333	10.8828	-0.1276	0.3464	-0.1667	-0.1172	0.3233
{10, 12, 13, 16}	{0, 3, 5, 8}	0.0422	3.1986	4.5844	8.1791	0.0422	0.1986	-0.4156	0.1791	0.3228
{11, 12, 13, 14}	{0, 2, 3, 4}	0.1145	1.6208	3.0066	4.2896	0.1145	-0.3792	0.0066	0.2896	0.3226
{9, 13, 14, 16}	{0, 6, 8, 10}	-0.0021	6.3641	7.6471	9.9588	-0.0021	0.3641	-0.3529	-0.0412	0.3220
{10, 12, 14, 15}	{0, 3, 6, 7}	-0.0057	3.1507	5.8194	7.0138	-0.0057	0.1507	-0.1806	0.0138	0.3214
{10, 11, 14, 17}	{0, 2, 6, 9}	0.1038	1.7539	5.9289	9.2902	0.1038	-0.2461	-0.0711	0.2902	0.3212
{10, 11, 13, 19}	{0, 2, 5, 11}	0.1908	1.8408	4.7329	11.3028	0.1908	-0.1592	-0.2671	0.3028	0.3205
{10, 11, 15, 16}	{0, 2, 7, 8}	0.0686	1.7186	7.0881	8.2055	0.0686	-0.2814	0.0881	0.2055	0.3201
{9, 12, 15, 17}	{0, 5, 9, 11}	0.0358	5.0162	8.8793	11.0462	0.0358	0.0162	-0.1207	0.0462	0.3199
{10, 12, 13, 17}	{0, 3, 5, 9}	0.0352	3.1916	4.5773	9.2216	0.0352	0.1916	-0.4227	0.2216	0.3191
{9, 13, 14, 17}	{0, 6, 8, 11}	-0.0117	6.3545	7.6375	10.9988	-0.0117	0.3545	-0.3625	-0.0012	0.3183
{10, 11, 14, 18}	{0, 2, 6, 10}	0.1086	1.7587	5.9337	10.2846	0.1086	-0.2413	-0.0663	0.2846	0.3179
{11, 12, 13, 15}	{0, 2, 3, 5}	0.0780	1.5844	2.9701	5.4475	0.0780	-0.4156	-0.0299	0.4475	0.3178
{9, 13, 15, 16}	{0, 6, 9, 10}	-0.0482	6.3180	8.7954	9.9127	-0.0482	0.3180	-0.2046	-0.0873	0.3172
{10, 11, 15, 17}	{0, 2, 7, 9}	0.0618	1.7118	7.0813	9.2482	0.0618	-0.2882	0.0813	0.2482	0.3164
{10, 12, 13, 18}	{0, 3, 5, 10}	0.0393	3.1957	4.5814	10.2153	0.0393	0.1957	-0.4186	0.2153	0.3158
{9, 12, 16, 17}	{0, 5, 10, 11}	0.0110	4.9914	9.9719	11.0214	0.0110	-0.0086	-0.0281	0.0214	0.3158
{10, 13, 14, 15}	{0, 5, 6, 7}	0.1473	4.6895	5.9724	7.1669	0.1473	-0.3105	-0.0276	0.1669	0.3150
{10, 11, 14, 19}	{0, 2, 6, 11}	0.1220	1.7720	5.9471	11.2339	0.1220	-0.2280	-0.0529	0.2339	0.3150
{11, 12, 13, 16}	{0, 2, 3, 7}	0.0606	1.5670	2.9527	6.5474	0.0606	-0.4330	-0.0473	-0.4526	0.3137
{10, 12, 14, 17}	{0, 3, 6, 9}	-0.0367	3.1197	5.7884	9.1497	-0.0367	0.1197	-0.2116	0.1497	0.3136
{9, 13, 15, 17}	{0, 6, 9, 11}	-0.0585	6.3076	8.7850	10.9519	-0.0585	0.3076	-0.2150	-0.0481	0.3135
{10, 11, 15, 18}	{0, 2, 7, 10}	0.0662	1.7163	7.0858	10.2422	0.0662	-0.2837	0.0858	0.2422	0.3131
{10, 12, 13, 19}	{0, 3, 5, 11}	0.0521	3.2085	4.5942	11.1641	0.0521	0.2085	-0.4058	0.1641	0.3129
{10, 12, 15, 16}	{0, 3, 7, 8}	-0.0733	3.0832	6.9463	8.0636	-0.0733	0.0832	-0.0537	0.0636	0.3125
{11, 12, 14, 15}	{0, 2, 4, 5}	0.0128	1.5192	4.1879	5.3823	0.0128	-0.4808	0.1879	0.3823	0.3123
{10, 11, 16, 17}	{0, 2, 8, 9}	0.0394	1.6894	8.1762	9.2258	0.0394	-0.3106	0.1762	0.2258	0.3122
{9, 14, 15, 16}	{0, 8, 9, 10}	0.1217	7.7708	8.9653	10.0826	0.1217	-0.2292	-0.0347	0.0826	0.3117
{10, 13, 14, 16}	{0, 5, 6, 8}	0.1260	4.6681	5.9511	8.2628	0.1260	-0.3319	-0.0489	0.2628	0.3109
{10, 11, 15, 19}	{0, 2, 7, 11}	0.0794	1.7294	7.0989	11.1913	0.0794	-0.2706	0.0989	0.1913	0.3102
{11, 12, 13, 17}	{0, 2, 3, 8}	0.2475	1.7538	3.1396	7.7838	0.2475	-0.2462	0.1396	-0.2162	0.3100
{9, 13, 16, 17}	{0, 6, 10, 11}	-0.0851	6.2810	9.8758	10.9253	-0.0851	0.2810	-0.1242	-0.0747	0.3094
{10, 11, 16, 18}	{0, 2, 8, 10}	0.0436	1.6937	8.1805	10.2196	0.0436	-0.3063	0.1805	0.2196	0.3090
{10, 12, 15, 17}	{0, 3, 7, 9}	-0.0819	3.0745	6.9376	9.1045	-0.0819	0.0745	-0.0624	0.1045	0.3088

Harmonic State	Closest Pitch Set	Tuned First Pitch	Tuned Second Pitch	Tuned Third Pitch	Tuned Fourth Pitch	First Pitch Adjustment	Second Pitch Adjustment	Third Pitch Adjustment	Fourth Pitch Adjustment	Tonalness
{11, 12, 14, 16}	{0, 2, 4, 6}	-0.0058	1.5005	4.1693	6.4810	-0.0058	-0.4995	0.1693	0.4810	0.3082
{9, 14, 15, 17}	{0, 8, 9, 11}	0.1132	7.7624	8.9568	11.1237	0.1132	-0.2376	-0.0432	0.1237	0.3080
{10, 12, 14, 19}	{0, 3, 6, 11}	-0.0209	3.1355	5.8042	11.0911	-0.0209	0.1355	-0.1958	0.0911	0.3074
{10, 13, 14, 17}	{0, 5, 6, 9}	0.1196	4.6618	5.9447	9.3060	0.1196	-0.3382	-0.0553	0.3060	0.3072
{11, 12, 13, 18}	{0, 2, 3, 9}	0.2470	1.7534	3.1391	8.7730	0.2470	-0.2466	0.1391	-0.2270	0.3067
{10, 13, 15, 16}	{0, 5, 7, 8}	0.0829	4.6250	7.1024	8.2197	0.0829	-0.3750	0.1024	0.2197	0.3061
{10, 11, 16, 19}	{0, 2, 8, 11}	0.0567	1.7068	8.1936	11.1687	0.0567	-0.2932	0.1936	0.1687	0.3060
{11, 13, 14, 15}	{0, 3, 4, 5}	-0.0943	2.7978	4.0808	5.2752	-0.0943	-0.2022	0.0808	0.2752	0.3059
{10, 12, 15, 18}	{0, 3, 7, 10}	-0.0789	3.0775	6.9406	10.0970	-0.0789	0.0775	-0.0594	0.0970	0.3056
{10, 11, 17, 18}	{0, 2, 9, 10}	0.0363	1.6863	9.2227	10.2122	0.0363	-0.3137	0.2227	0.2122	0.3053
{10, 12, 16, 17}	{0, 3, 8, 9}	-0.1069	3.0496	8.0300	9.0796	-0.1069	0.0496	0.0300	0.0796	0.3047
{11, 12, 14, 17}	{0, 2, 4, 8}	0.1836	1.6900	4.3587	7.7200	0.1836	-0.3100	0.3587	-0.2800	0.3045
{10, 13, 14, 18}	{0, 5, 6, 10}	0.1248	4.6670	5.9499	10.3008	0.1248	-0.3330	-0.0501	0.3008	0.3039
{9, 14, 16, 17}	{0, 8, 10, 11}	0.0885	7.7376	10.0494	11.0989	0.0885	-0.2624	0.0494	0.0989	0.3039
{11, 12, 13, 19}	{0, 2, 3, 10}	0.0827	1.5891	2.9748	9.5446	0.0827	-0.4109	-0.0252	-0.4554	0.3038
{11, 12, 15, 16}	{0, 1, 5, 6}	-0.0459	1.4605	5.3236	6.4409	-0.0459	0.4605	0.3236	0.4409	0.3034
{10, 12, 15, 19}	{0, 3, 7, 11}	-0.0669	3.0896	6.9527	11.0451	-0.0669	0.0896	-0.0473	0.0451	0.3026
{10, 13, 15, 17}	{0, 5, 7, 9}	0.0759	4.6180	7.0954	9.2623	0.0759	-0.3820	0.0954	0.2623	0.3024
{10, 11, 17, 19}	{0, 2, 9, 11}	0.0495	1.6995	9.2359	11.1615	0.0495	-0.3005	0.2359	0.1615	0.3024
{11, 13, 14, 16}	{0, 3, 4, 6}	-0.1148	2.7773	4.0603	6.3721	-0.1148	-0.2227	0.0603	0.3721	0.3018
{11, 12, 14, 18}	{0, 2, 4, 9}	0.1825	1.6889	4.3576	8.7084	0.1825	-0.3111	0.3576	-0.2916	0.3012
{11, 12, 13, 20}	{0, 2, 3, 10}	0.1060	1.6124	2.9981	10.4560	0.1060	-0.3876	-0.0019	0.4560	0.3012
{10, 13, 14, 19}	{0, 5, 6, 11}	0.1389	4.6811	5.9641	11.2509	0.1389	-0.3189	-0.0359	0.2509	0.3010
{10, 14, 15, 16}	{0, 6, 7, 8}	0.0088	5.8339	7.0283	8.1456	0.0088	-0.1661	0.0283	0.1456	0.3006
{11, 12, 15, 17}	{0, 2, 6, 8}	0.1460	1.6524	5.5156	7.6824	0.1460	-0.3476	-0.4844	-0.3176	0.2997
{10, 13, 15, 18}	{0, 5, 7, 10}	0.0807	4.6228	7.1002	10.2567	0.0807	-0.3772	0.1002	0.2567	0.2991
{9, 15, 16, 17}	{0, 9, 10, 11}	0.0410	8.8846	10.0019	11.0514	0.0410	-0.1154	0.0019	0.0514	0.2991
{10, 11, 18, 19}	{0, 2, 10, 11}	0.0540	1.7040	10.2299	11.1660	0.0540	-0.2960	0.2299	0.1660	0.2991
{11, 12, 13, 21}	{0, 2, 3, 11}	0.1344	1.6408	3.0265	11.3291	0.1344	-0.3592	0.0265	0.3291	0.2988
{10, 12, 16, 19}	{0, 3, 8, 11}	-0.0921	3.0643	8.0448	11.0199	-0.0921	0.0643	0.0448	0.0199	0.2985
{12, 13, 14, 15}	{0, 1, 3, 4}	0.0104	1.3962	2.6792	3.8736	0.0104	0.3962	-0.3208	-0.1264	0.2984
{11, 12, 14, 19}	{0, 2, 4, 9}	0.0145	1.5208	4.1896	9.4764	0.0145	-0.4792	0.1896	0.4764	0.2983
{10, 13, 16, 17}	{0, 5, 8, 9}	0.0526	4.5948	8.1895	9.2391	0.0526	-0.4052	0.1895	0.2391	0.2982
{11, 13, 14, 17}	{0, 3, 4, 8}	0.0774	2.9695	4.2525	7.6138	0.0774	-0.0305	0.2525	-0.3862	0.2981
{10, 12, 17, 18}	{0, 3, 9, 10}	-0.1135	3.0430	9.0730	10.0625	-0.1135	0.0430	0.0730	0.0625	0.2977
{11, 13, 15, 16}	{0, 3, 5, 6}	-0.1574	2.7347	5.2121	6.3294	-0.1574	-0.2653	0.2121	0.3294	0.2970
{10, 14, 15, 17}	{0, 6, 7, 9}	0.0007	5.8259	7.0203	9.1872	0.0007	-0.1741	0.0203	0.1872	0.2969