

# Tuning Three Pitches

Just intonation occurs when all the pitches of a chord are tuned to be harmonics of a common fundamental frequency. Tuning in this way makes a chord “ring” without any beats. The chart below reveals how to tune many common three pitch chords without any pitch drift. It achieves this by preserving the intonation of the reinforced fundamental of the chord. It lists the **closest pitch set**, **tuned pitches**, **pitch adjustments**, and **tonalness values** for the most tunable three-pitch harmonic states.

A **harmonic state** is a set of partial numbers above a common fundamental frequency. For example, the harmonic state {4,5,6} represents the fourth, fifth, and sixth harmonics of a given fundamental. In other words, the harmonic state {4,5,6} is a justly tuned major triad.

The **closest pitch set** is a set of semitone values that best approximates the harmonic state with pitches from the equal tempered chromatic scale. For example, the pitch set {0,4,7} represents two pitches which are four and seven semitones above a lowest pitch. In other words, the pitch set {0,4,7} is an equal tempered major triad.

The **tuned pitches** are a set of semitone values which represent how to properly tune the chord and preserve its reinforced fundamental.

The **pitch adjustments** are simply the difference between the tuned pitches and the closest pitch set. They reveal how to tune each of the pitches of the chord relative to equal temperament.

The **tonalness values** reveal the strength of the reinforced fundamental frequency. For this chart, the tonalness calculation assumes that each component pitch of the chord has an amplitude equal to 1. A higher tonalness value means that the chord will “ring” more if it is properly tuned.

Harmonic State	Closest Pitch Set	Tuned Lower Pitch	Tuned Middle Pitch	Tuned Upper Pitch	Lower Pitch Adjustment	Middle Pitch Adjustment	Upper Pitch Adjustment	Tonalness
{3, 4, 5}	{0, 5, 9}	0.0462	5.0266	8.8898	0.0462	0.0266	-0.1102	0.7833
{4, 5, 6}	{0, 4, 7}	0.0391	3.9022	7.0587	0.0391	-0.0978	0.0587	0.6167
{4, 5, 7}	{0, 4, 10}	0.1213	3.9844	9.8095	0.1213	-0.0156	-0.1905	0.5929
{4, 6, 7}	{0, 7, 10}	0.0738	7.0933	9.7620	0.0738	0.0933	-0.2380	0.5595
{5, 6, 7}	{0, 3, 6}	-0.0021	3.1543	5.8230	-0.0021	0.1543	-0.1770	0.5095
{5, 6, 8}	{0, 3, 8}	-0.0878	3.0686	8.0490	-0.0878	0.0686	0.0490	0.4917
{5, 6, 9}	{0, 3, 10}	-0.0955	3.0609	10.0805	-0.0955	0.0609	0.0805	0.4778
{5, 7, 8}	{0, 6, 8}	0.0168	5.8420	8.1537	0.0168	-0.1580	0.1537	0.4679
{5, 7, 9}	{0, 6, 10}	0.0120	5.8371	10.1879	0.0120	-0.1629	0.1879	0.4540
{5, 8, 9}	{0, 8, 10}	-0.0841	8.0528	10.0919	-0.0841	0.0528	0.0919	0.4361
{6, 7, 8}	{0, 3, 5}	0.1145	2.7833	5.0950	0.1145	-0.2167	0.0950	0.4345
{6, 7, 9}	{0, 3, 7}	0.1073	2.7761	7.1269	0.1073	-0.2239	0.1269	0.4206
{6, 7, 10}	{0, 3, 9}	0.1538	2.8225	8.9973	0.1538	-0.1775	-0.0027	0.4095

Harmonic State	Closest Pitch Set	Tuned Lower Pitch	Tuned Middle Pitch	Tuned Upper Pitch	Lower Pitch Adjustment	Middle Pitch Adjustment	Upper Pitch Adjustment	Tonalness
{6, 8, 9}	{0, 5, 7}	0.0007	4.9811	7.0202	0.0007	-0.0189	0.0202	0.4028
{6, 7, 11}	{0, 3, 10}	0.0061	2.6748	10.4998	0.0061	-0.3252	0.4998	0.4004
{6, 8, 11}	{0, 5, 10}	-0.1109	4.8695	10.3827	-0.1109	-0.1305	0.3827	0.3826
{7, 8, 9}	{0, 2, 4}	-0.2057	2.1061	4.1452	-0.2057	0.1061	0.1452	0.3790
{6, 9, 10}	{0, 7, 9}	0.0357	7.0552	8.8792	0.0357	0.0552	-0.1208	0.3778
{6, 9, 11}	{0, 7, 10}	-0.1276	6.8919	10.3660	-0.1276	-0.1081	0.3660	0.3687
{7, 8, 10}	{0, 2, 6}	-0.1535	2.1583	6.0214	-0.1535	0.1583	0.0214	0.3679
{7, 8, 11}	{0, 2, 8}	-0.0643	2.2475	7.7607	-0.0643	0.2475	-0.2393	0.3588
{6, 10, 11}	{0, 9, 10}	-0.0818	8.7618	10.4119	-0.0818	-0.2382	0.4119	0.3576
{7, 9, 10}	{0, 4, 6}	-0.1595	4.1913	6.0153	-0.1595	0.1913	0.0153	0.3540
{7, 8, 12}	{0, 2, 9}	-0.1896	2.1222	9.1417	-0.1896	0.1222	0.1417	0.3512
{7, 9, 11}	{0, 4, 8}	-0.0669	4.2840	7.7580	-0.0669	0.2840	-0.2420	0.3449
{7, 8, 13}	{0, 2, 11}	-0.0499	2.2619	10.6671	-0.0499	0.2619	-0.3329	0.3448
{7, 9, 12}	{0, 4, 9}	-0.1974	4.1534	9.1339	-0.1974	0.1534	0.1339	0.3373
{8, 9, 10}	{0, 2, 4}	0.0278	2.0669	3.8909	0.0278	0.0669	-0.1091	0.3361
{7, 10, 11}	{0, 6, 8}	-0.0047	6.1702	7.8202	-0.0047	0.1702	-0.1798	0.3338
{7, 9, 13}	{0, 4, 11}	-0.0520	4.2988	10.6650	-0.0520	0.2988	-0.3350	0.3309
{8, 9, 11}	{0, 2, 6}	0.1220	2.1611	5.6352	0.1220	0.1611	-0.3648	0.3270
{7, 10, 12}	{0, 6, 9}	-0.1382	6.0366	9.1930	-0.1382	0.0366	0.1930	0.3262
{7, 10, 13}	{0, 6, 11}	0.0134	6.1883	10.7304	0.0134	0.1883	-0.2696	0.3198
{8, 9, 12}	{0, 2, 7}	-0.0187	2.0204	7.0009	-0.0187	0.0204	0.0009	0.3194
{7, 11, 12}	{0, 8, 9}	-0.0369	7.7881	9.2944	-0.0369	-0.2119	0.2944	0.3171
{8, 10, 11}	{0, 4, 6}	0.1834	4.0466	5.6966	0.1834	0.0466	-0.3034	0.3159
{8, 9, 13}	{0, 2, 8}	-0.1135	1.9256	8.2918	-0.1135	-0.0744	0.2918	0.3130
{7, 11, 13}	{0, 8, 11}	0.1213	7.9462	10.8383	0.1213	-0.0538	-0.1617	0.3107
{8, 9, 14}	{0, 2, 10}	0.0583	2.0974	9.7465	0.0583	0.0974	-0.2535	0.3075
{7, 12, 13}	{0, 9, 11}	-0.0193	9.3120	10.6978	-0.0193	0.3120	-0.3022	0.3031
{8, 9, 15}	{0, 2, 11}	0.0115	2.0506	10.8942	0.0115	0.0506	-0.1058	0.3028
{9, 10, 11}	{0, 2, 3}	-0.0844	1.7396	3.3896	-0.0844	-0.2604	0.3896	0.3020
{8, 10, 13}	{0, 4, 8}	-0.0579	3.8052	8.3474	-0.0579	-0.1948	0.3474	0.3019
{8, 11, 12}	{0, 6, 7}	0.1425	5.6556	7.1620	0.1425	-0.3444	0.1620	0.2992
{9, 10, 12}	{0, 2, 5}	0.0653	1.8893	5.0457	0.0653	-0.1107	0.0457	0.2944
{8, 11, 13}	{0, 6, 8}	0.0447	5.5579	8.4499	0.0447	-0.4421	0.4499	0.2928
{8, 10, 15}	{0, 4, 11}	0.0737	3.9369	10.9564	0.0737	-0.0631	-0.0436	0.2917
{9, 10, 13}	{0, 2, 6}	-0.0367	1.7873	6.3295	-0.0367	-0.2127	0.3295	0.2880
{8, 11, 14}	{0, 6, 10}	0.2315	5.7447	9.9198	0.2315	-0.2553	-0.0802	0.2873

Harmonic State	Closest Pitch Set	Tuned Lower Pitch	Tuned Middle Pitch	Tuned Upper Pitch	Lower Pitch Adjustment	Middle Pitch Adjustment	Upper Pitch Adjustment	Tonalness
{9, 11, 12}	{0, 3, 5}	-0.1453	3.3288	4.8351	-0.1453	0.3288	-0.1649	0.2854
{8, 12, 13}	{0, 7, 8}	-0.1150	6.9046	8.2903	-0.1150	-0.0954	0.2903	0.2853
{8, 11, 15}	{0, 6, 11}	0.1843	5.6975	11.0670	0.1843	-0.3025	0.0670	0.2826
{9, 10, 14}	{0, 2, 8}	0.1510	1.9750	7.8001	0.1510	-0.0250	-0.1999	0.2825
{9, 11, 13}	{0, 3, 6}	-0.2555	3.2186	6.1107	-0.2555	0.2186	0.1107	0.2789
{9, 10, 15}	{0, 2, 9}	0.1009	1.9249	8.9445	0.1009	-0.0751	-0.0555	0.2778
{8, 12, 15}	{0, 7, 11}	0.0225	7.0421	10.9052	0.0225	0.0421	-0.0948	0.2750
{10, 11, 12}	{0, 2, 3}	0.0685	1.7185	3.2249	0.0685	-0.2815	0.2249	0.2742
{9, 10, 16}	{0, 2, 10}	0.0732	1.8973	10.0341	0.0732	-0.1027	0.0341	0.2736
{9, 11, 14}	{0, 3, 8}	-0.0660	3.4081	7.5832	-0.0660	0.4081	-0.4168	0.2734
{8, 13, 14}	{0, 8, 10}	-0.0326	8.3727	9.6557	-0.0326	0.3727	-0.3443	0.2734
{9, 12, 13}	{0, 5, 6}	-0.0978	4.8827	6.2684	-0.0978	-0.1173	0.2684	0.2714
{9, 10, 17}	{0, 2, 11}	0.0629	1.8869	11.0734	0.0629	-0.1131	0.0734	0.2699
{9, 11, 15}	{0, 3, 9}	-0.1216	3.3525	8.7220	-0.1216	0.3525	-0.2780	0.2687
{8, 13, 15}	{0, 8, 11}	-0.0870	8.3183	10.7957	-0.0870	0.3183	-0.2043	0.2686
{10, 11, 13}	{0, 2, 5}	0.2503	1.9003	4.7924	0.2503	-0.0997	-0.2076	0.2678
{9, 12, 14}	{0, 5, 8}	0.1004	5.0808	7.7495	0.1004	0.0808	-0.2505	0.2659
{9, 11, 16}	{0, 3, 10}	-0.1537	3.3204	9.8072	-0.1537	0.3204	-0.1928	0.2645
{8, 14, 15}	{0, 10, 11}	0.1144	9.8026	10.9970	0.1144	-0.1974	-0.0030	0.2631
{10, 11, 14}	{0, 2, 6}	0.1689	1.8189	5.9940	0.1689	-0.1811	-0.0060	0.2623
{9, 11, 17}	{0, 3, 11}	-0.1676	3.3065	10.8429	-0.1676	0.3065	-0.1571	0.2608
{10, 12, 13}	{0, 3, 5}	0.0852	3.2417	4.6274	0.0852	0.2417	-0.3726	0.2603
{9, 13, 14}	{0, 6, 8}	-0.0120	6.3542	7.6372	-0.0120	0.3542	-0.3628	0.2595
{10, 11, 15}	{0, 2, 7}	0.1185	1.7685	7.1380	0.1185	-0.2315	0.1380	0.2576
{9, 12, 16}	{0, 5, 10}	0.0159	4.9963	9.9768	0.0159	-0.0037	-0.0232	0.2569
{9, 13, 15}	{0, 6, 9}	-0.0696	6.2965	8.7739	-0.0696	0.2965	-0.2261	0.2547
{10, 11, 16}	{0, 2, 8}	0.0918	1.7418	8.2287	0.0918	-0.2582	0.2287	0.2534
{9, 12, 17}	{0, 5, 11}	0.0040	4.9845	11.0145	0.0040	-0.0155	0.0145	0.2533
{11, 12, 13}	{0, 2, 3}	0.1968	1.7032	3.0889	0.1968	-0.2968	0.0889	0.2512
{9, 13, 16}	{0, 6, 10}	-0.1027	6.2635	9.8582	-0.1027	0.2635	-0.1418	0.2505
{10, 12, 15}	{0, 3, 7}	-0.0574	3.0991	6.9622	-0.0574	0.0991	-0.0378	0.2500
{10, 11, 17}	{0, 2, 9}	0.0835	1.7335	9.2699	0.0835	-0.2665	0.2699	0.2497
{9, 14, 15}	{0, 8, 9}	0.1424	7.7916	8.9860	0.1424	-0.2084	-0.0140	0.2492
{10, 13, 14}	{0, 5, 6}	0.1921	4.7343	6.0172	0.1921	-0.2657	0.0172	0.2484
{9, 13, 17}	{0, 6, 11}	-0.1166	6.2496	10.8939	-0.1166	0.2496	-0.1061	0.2469
{10, 11, 18}	{0, 2, 10}	0.0894	1.7395	10.2654	0.0894	-0.2605	0.2654	0.2465

Harmonic State	Closest Pitch Set	Tuned Lower Pitch	Tuned Middle Pitch	Tuned Upper Pitch	Lower Pitch Adjustment	Middle Pitch Adjustment	Upper Pitch Adjustment	Tonalness
{11, 12, 14}	{0, 2, 4}	0.1165	1.6229	4.2916	0.1165	-0.3771	0.2916	0.2457
{9, 14, 16}	{0, 8, 10}	0.1122	7.7614	10.0731	0.1122	-0.2386	0.0731	0.2450
{10, 13, 15}	{0, 5, 7}	0.1392	4.6814	7.1588	0.1392	-0.3186	0.1588	0.2436
{10, 11, 19}	{0, 2, 11}	0.1064	1.7565	11.2184	0.1064	-0.2435	0.2184	0.2435
{10, 12, 17}	{0, 3, 9}	-0.0991	3.0573	9.0873	-0.0991	0.0573	0.0873	0.2422
{9, 14, 17}	{0, 8, 11}	0.1013	7.7504	11.1117	0.1013	-0.2496	0.1117	0.2414
{11, 12, 15}	{0, 2, 5}	0.0685	1.5749	5.4380	0.0685	-0.4251	0.4380	0.2409
{9, 15, 16}	{0, 9, 10}	0.0536	8.8972	10.0145	0.0536	-0.1028	0.0145	0.2403
{10, 13, 16}	{0, 5, 8}	0.1114	4.6535	8.2482	0.1114	-0.3465	0.2482	0.2394
{11, 13, 14}	{0, 3, 4}	-0.0176	2.8745	4.1575	-0.0176	-0.1255	0.1575	0.2393
{10, 14, 15}	{0, 6, 7}	0.0470	5.8721	7.0665	0.0470	-0.1279	0.0665	0.2381
{11, 12, 16}	{0, 2, 7}	0.0452	1.5516	6.5321	0.0452	-0.4484	-0.4679	0.2367
{9, 15, 17}	{0, 9, 11}	0.0415	8.8851	11.0519	0.0415	-0.1149	0.0519	0.2366
{10, 12, 19}	{0, 3, 11}	-0.0802	3.0762	11.0318	-0.0802	0.0762	0.0318	0.2360
{10, 13, 17}	{0, 5, 9}	0.1029	4.6450	9.2893	0.1029	-0.3550	0.2893	0.2357