

Tuning Two Pitches

Just intonation occurs when all the pitches of a chord are tuned to be harmonics of a common fundamental frequency. Tuning in this way makes a chord “ring” without any beats. The chart below reveals how to tune many common two pitch chords without any pitch drift. It achieves this by preserving the intonation of the reinforced fundamental of the chord. It lists the **closest pitch set**, **tuned pitches**, **pitch adjustments**, and **tonalness values** for the most tunable two-pitch harmonic states.

A **harmonic state** is a set of partial numbers above a common fundamental frequency. For example, the harmonic state {4,5} represents the fourth and fifth harmonics of a given fundamental. In other words, the harmonic state {4,5} is a justly tuned major third.

The **closest pitch set** is a set of semitone values that best approximates the harmonic state with pitches from the equal tempered chromatic scale. For example, the pitch set {0,4} represents two pitches which are four semitones apart. In other words, the pitch set {0,4} is an equal tempered major third.

The **tuned pitches** are a set of semitone values which represent how to properly tune the chord and preserve its reinforced fundamental.

The **pitch adjustments** are simply the difference between the tuned pitches and the closest pitch set. The reveal how to tune each of the pitches of the chord relative to equal temperament.

The **tonalness values** reveal the strength of the reinforced fundamental frequency. For this chart, the tonalness calculation assumes that each component pitch of the chord has an amplitude equal to 1. A higher tonalness value means that the chord will “ring” more if it is properly tuned.

Harmonic State	Closest Pitch Set	Tuned Lower Pitch	Tuned Upper Pitch	Lower Pitch Adjustment	Upper Pitch Adjustment	Tonalness
{1, 1}	{0, 0}	0.0000	0.0000	0.0000	0.0000	2.0000
{1, 2}	{0, 12}	0.0000	12.0000	0.0000	0.0000	1.5000
{2, 3}	{0, 7}	-0.0078	7.0117	-0.0078	0.0117	0.8333
{3, 4}	{0, 5}	0.0084	4.9888	0.0084	-0.0112	0.5833
{3, 5}	{0, 9}	0.0587	8.9022	0.0587	-0.0978	0.5333
{4, 5}	{0, 4}	0.0608	3.9240	0.0608	-0.0760	0.4500
{4, 7}	{0, 10}	0.1134	9.8016	0.1134	-0.1984	0.3929
{5, 6}	{0, 3}	-0.0711	3.0853	-0.0711	0.0853	0.3667
{5, 7}	{0, 6}	0.0729	5.8980	0.0729	-0.1020	0.3429
{5, 8}	{0, 8}	-0.0526	8.0842	-0.0526	0.0842	0.3250
{5, 9}	{0, 10}	-0.0628	10.1131	-0.0628	0.1131	0.3111
{6, 7}	{0, 3}	0.1529	2.8216	0.1529	-0.1784	0.3095
{7, 8}	{0, 2}	-0.1455	2.1663	-0.1455	0.1663	0.2679
{6, 11}	{0, 10}	-0.1742	10.3194	-0.1742	0.3194	0.2576
{7, 9}	{0, 4}	-0.1535	4.1973	-0.1535	0.1973	0.2540
{7, 10}	{0, 6}	-0.0720	6.1029	-0.0720	0.1029	0.2429
{8, 9}	{0, 2}	-0.0184	2.0207	-0.0184	0.0207	0.2361
{7, 11}	{0, 8}	0.0681	7.8930	0.0681	-0.1070	0.2338
{7, 12}	{0, 9}	-0.1221	9.2092	-0.1221	0.2092	0.2262

Harmonic State	Closest Pitch Set	Tuned Lower Pitch	Tuned Upper Pitch	Lower Pitch Adjustment	Upper Pitch Adjustment	Tonalness
{7, 13}	{0, 11}	0.0990	10.8161	0.0990	-0.1839	0.2198
{8, 11}	{0, 6}	0.2050	5.7182	0.2050	-0.2818	0.2159
{9, 10}	{0, 2}	0.0834	1.9074	0.0834	-0.0926	0.2111
{9, 11}	{0, 3}	-0.2133	3.2607	-0.2133	0.2607	0.2020
{8, 13}	{0, 8}	-0.1544	8.2509	-0.1544	0.2509	0.2019
{8, 15}	{0, 11}	0.0408	10.9235	0.0408	-0.0765	0.1917
{10, 11}	{0, 2}	0.1666	1.8167	0.1666	-0.1833	0.1909
{9, 13}	{0, 6}	-0.1498	6.2164	-0.1498	0.2164	0.1880
{9, 14}	{0, 8}	0.1373	7.7864	0.1373	-0.2136	0.1825
{10, 13}	{0, 5}	0.1991	4.7412	0.1991	-0.2588	0.1769
{11, 12}	{0, 2}	0.2361	1.7425	0.2361	-0.2575	0.1742
{9, 16}	{0, 10}	0.0141	9.9750	0.0141	-0.0250	0.1736
{9, 17}	{0, 11}	-0.0036	11.0068	-0.0036	0.0068	0.1699
{11, 13}	{0, 3}	0.0495	2.9416	0.0495	-0.0584	0.1678
{11, 14}	{0, 4}	-0.0770	4.0980	-0.0770	0.0980	0.1623
{12, 13}	{0, 1}	-0.1851	1.2006	-0.1851	0.2006	0.1603
{10, 17}	{0, 9}	-0.0690	9.1174	-0.0690	0.1174	0.1588
{11, 15}	{0, 5}	-0.1563	5.2132	-0.1563	0.2132	0.1576
{11, 16}	{0, 6}	-0.1983	6.2885	-0.1983	0.2885	0.1534
{10, 19}	{0, 11}	-0.0386	11.0734	-0.0386	0.0734	0.1526
{11, 17}	{0, 8}	0.1821	7.7185	0.1821	-0.2815	0.1497
{13, 14}	{0, 1}	-0.1363	1.1467	-0.1363	0.1467	0.1484
{11, 18}	{0, 9}	0.1798	8.7057	0.1798	-0.2943	0.1465
{13, 15}	{0, 2}	-0.2217	2.2558	-0.2217	0.2558	0.1436
{11, 19}	{0, 9}	-0.1694	9.2926	-0.1694	0.2926	0.1435
{12, 17}	{0, 6}	-0.0124	6.0176	-0.0124	0.0176	0.1422
{11, 20}	{0, 10}	-0.1242	10.2258	-0.1242	0.2258	0.1409
{13, 16}	{0, 4}	0.1817	3.7764	0.1817	-0.2236	0.1394
{11, 21}	{0, 11}	-0.0669	11.1277	-0.0669	0.1277	0.1385
{14, 15}	{0, 1}	-0.0939	1.1006	-0.0939	0.1006	0.1381
{12, 19}	{0, 8}	0.0172	7.9728	0.0172	-0.0272	0.1360